Abstract: Polynomial interpolation problems have been largely studied in algebraic geometry and commutative algebra. The classical question asks what is the dimension of a linear system of hypersurfaces in $\mathbb{P}^n$ of given degree and with multiple base points. The case of general double points has a very long history, which goes back to the classical school of algebraic geometry of the XIX century. However, it has been completely solved by J. Alexander and A. Hirschowitz only in 1995, by using the so-called méthode d'Horace différentiel. For higher multiplicities of the base points, even the case of planar curves is in general open. A conjectural answer to the latter case is given by the Segre-Harbourne-Gimigliano-Hirschowitz Conjecture.

We consider a multi-graded version of the problem: what is the dimension of a linear system of curves in $\mathbb{P}^1 \times \mathbb{P}^1$ of given bidegree and with multiple base points? In 2005, M.V. Catalisano, A.V. Geramita and A. Gimigliano introduced the multiprojective-affine-projective method that reduces this problem to the classical case of fat points in $\mathbb{P}^2$ and solved the case of general double points.

After a general introduction, in this talk, I will present the solution to the case of triple points, and some partial result to higher multiplicities, by using a combination of the multiprojective-affine-projective method and the méthode d'Horace différentiel. If time permits, I will also explain how to use these ideas to study the Hilbert function of other types of 0-dimensional schemes and to prove that the tangential varieties to any Segre-Veronese embedding of $\mathbb{P}^1 \times \mathbb{P}^1$ are never defective.

These are joint works with E. Carlini and M. V. Catalisano.

Referenti: Alessandra Bernardi e Edoardo Ballico