How to use the nonlinearities to control a system

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A control system is a dynamical system on which one can act thanks to what is called the control. For example, in a car, one can turn the steering wheel, press the accelerator pedal etc. These are the control(s). One of the main problems in control theory is the controllability problem. It is the following one. One starts from a given situation and there is a given target. The controllability problem is to see if, by using some suitable controls depending on time, the given situation and target, one can move from the given situation to the target. We study this problem with a special emphasis on the case where the nonlinearities play a crucial role. In finite dimension in this case a key tool is the use of iterated Lie brackets as shown in particular by the Rashevski-Chow theorem. This key tool gives also important results for some control systems modeled by means of partial differential equations. However we do not know how to use it for many other control systems modeled by means of partial differential equations. We present methods to avoid the use of iterated Lie brackets. We give applications of these methods to the control of fluids modeled by various equations (Euler and Navier-Stokes equations of incompressible fluids, shallow water equations, Korteweg-de Vries equations).