Quantitative estimates for singular strata of minimizing maps between manifolds

Abstract:

Regularity properties for minimizing harmonic maps between Riemannian manifolds have been known since the classical work of Schoen and Uhlenbeck (1982); here an estimate on the Hausdorff dimension of the singular set $S(u)$ of a map $u$ is given: $\dim(S(u)) \leq n-3$, where $n$ is the dimension of the domain manifold. In this seminar, we are looking deeper into some more recent quantitative results, which describe precisely the structure of $S(u)$. As a starting point, we are recalling the main tools introduced in the work of Cheeger and Naber (2013), which allow to extend the previous estimate to the Minkowski dimension of $S(u)$: in particular, we are stratifying the singular set according to how close the map $u$ is, at any point of $S(u)$, to be invariant with respect to an affine plane. New techniques are then used to improve the aforesaid estimate: following the work of Naber and Valtorta (2017), we obtain an upper bound on the $(n-3)$-dimensional Minkowski content of $S(u)$. Furthermore, by the use of a suitable version of the Reifenberg Theorem, we manage to show that the above defined singular strata are rectifiable, thus gaining very powerful information on the structure of the singular set.

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