



UNIVERSITÀ  
DI TRENTO

Dipartimento di  
Matematica

# DOTTORATO



CYCLE 33th  
ORAL DEFENCE OF THE PHD THESIS

**Thursday 24 March 2022 – at 2:30 pm**  
**Seminar Room “-1”**

The event will take place online through the ZOOM platform.  
To get the access codes please contact the secretary office

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## Some questions regarding groups of automorphisms of primary trees

**Abstract:** Groups acting on  $p$ -adic trees have been well studied over the past decades since they represent a source of examples with interesting properties in group theory. Groups with intermediate growth or counterexamples to the General Burnside Problem can be found inside this class of groups. In this talk we analyze some properties concerning the structures of two families of groups acting over primary regular rooted trees. These two families are the GGS-groups acting over the  $p^n$ -adic tree and the  $p$ -Basilica groups, a generalization of the Basilica group over the  $p$ -adic tree for a prime  $p$ . For the GGS-groups over the  $p^n$ -adic tree we determine which of them are branch. We reduce our study to the fractal GGS-groups, since the non-fractal ones cannot be branch. We prove that all of them, except the ones acting over the  $2^n$ -adic tree whose defining vectors have only one invertible component in position  $2^{n-1}$ , are weakly regular branch. The GGS-groups with constant defining vector are weakly regular branch but not branch. For the other GGS-groups, we prove that they are all regular branch with some small exceptions for which the question is still open.

The  $p$ -Basilica groups are weakly branch but not branch for any prime  $p$ . These provide the first examples of groups with these properties that are super strongly fractal. For this class of groups we also study other problems. We show that they have the  $p$ -congruence subgroup property but not the congruence subgroup property nor the weak congruence subgroup property, providing the first examples of weakly branch groups with such properties. We compute the orders of the congruence quotients of these groups, which enables us to determine the Hausdorff dimensions of the  $p$ -Basilica groups. Lastly, we show that the  $p$ -Basilica groups do not possess maximal subgroups of infinite index and that they have infinitely many non-normal maximal subgroups.

**Supervisors:**

Andrea Caranti (University of Trento)

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**CONTATTI**

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