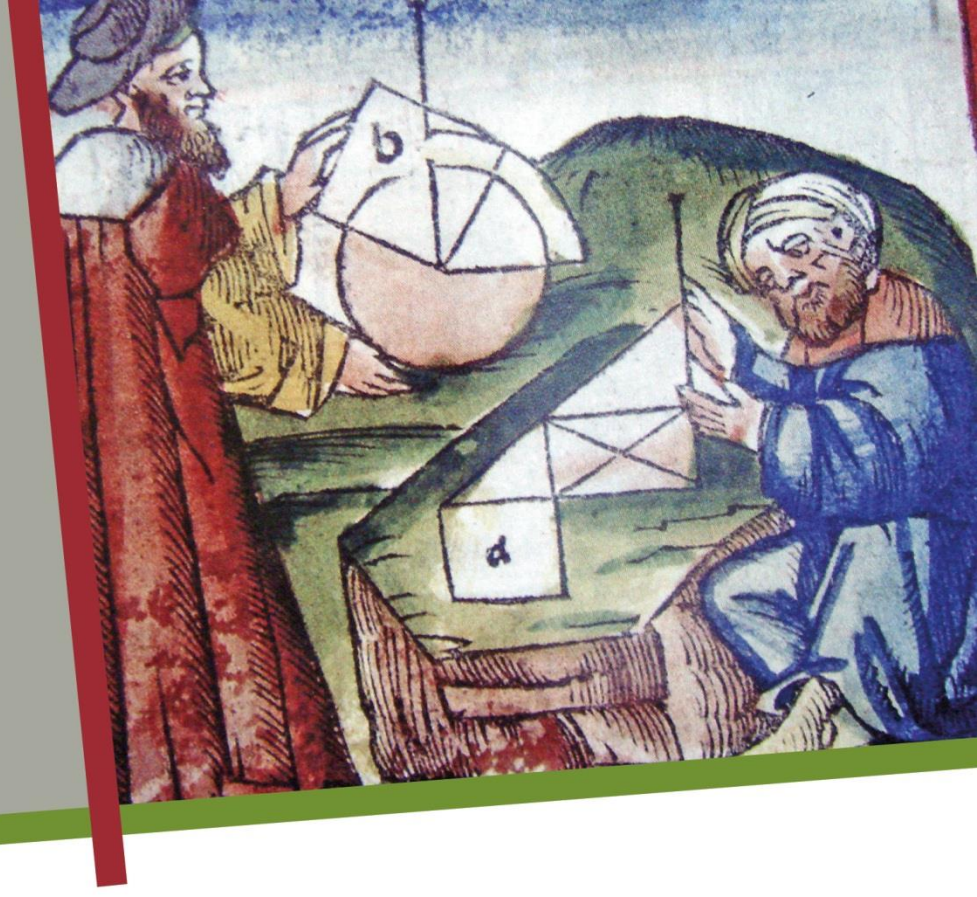
UNIVERSITÀ DEGLI STUDI
DI TRENTO

Dipartimento di Matematica




# SEMINARI

**Thursday 12 May – at 2.30 p.m.**

**Seminar Room "-1" – Povo 0**

## José Francisco Fernando Galvan

**(Universidad Complutense de Madrid)**

# Hilbert's 17th problem for global analytic functions

ABSTRACT. The representation of positive semidefinite analytic functions as sums of squares of meromorphic functions is an open problem (for dimension $\geq 3$) that arises from classical Hilbert's 17th problem (H17). Namely, *To determine if every positive semidefinite polynomial function on $\mathbb{R}^n$ is a sum of squares of rational functions on $\mathbb{R}^n$*. This problem was solved by E. Artin in 1927 using the theory of ordered fields. In the analytic setting the theory of orderings has not a good behavior and one has to explore other type of tools. For dimension $d \leq 2$, the problem is solved and one has the following:

- If $X$ is a global analytic surface and $f : X \to \mathbb{R}$ is a positive semidefinite analytic function, then $f$ is a sum of 10 squares of meromorphic functions on $X$ (Fernando).
- If $X$ is in addition coherent, then $f$ is a sum of 5 squares of meromorphic functions on $X$ (Acquistapace-Broglia-Fernando-Ruiz, Andradas-Díaz-Cano-Ruiz).
- If $X$ is in addition non-singular, then $f$ is a sum of 3 squares of analytic functions on $X$ (Jaworski).
- If $X$ is in addition not singular and unbounded, then $f$ is a sum of 2 squares of analytic functions on $X$ (Jaworski).

For $d \geq 3$, the problem is open even for $\mathbb{R}^d$ except for positive semidefinite analytic functions whose zero set outside a big enough ball is a discrete set. In this seminar we will talk about the reductions concerning this problem and some relevant consequences concerning its solution. Namely, *H17 has a solution if and only if every irreducible positive semidefinite analytic function whose zero set has dimension between 1 and $d-2$ is a sum of squares of meromorphic functions on a neighborhood of its zero set.*

A main consequence of H17 will be the *real analytic Nullstellensatz*, which is the natural amalgamation to the analytic setting of the classical real Nullstellensatz (Stengle) for the algebraic setting and Hilbert's Nullstellensatz for the complex analytic case (Forster).

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