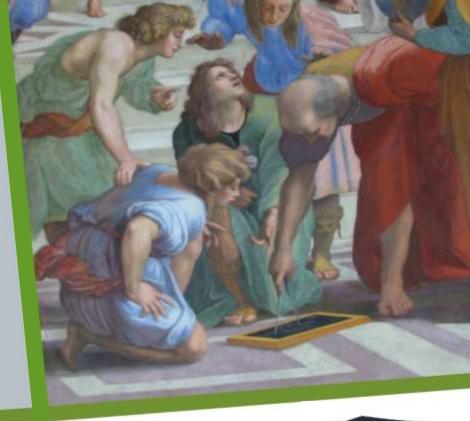


DOTORAU





CYCLE 34th ORAL DEFENCE OF THE PHD THESIS

Thursday 13 October 2022 – at 3:00 pm Seminar room "-1"

The event will take place in presence and online through the ZOOM platform. To get the access codes please contact the secretary office

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PhD Student in Mathematics

Local coherence of hearts in the derived category of a commutative ring

Abstract:

Approximation theory is a fundamental tool in order to study the representation theory of a ring R. Roughly speaking, it consists in determining suitable additive or abelian subcategories of the whole module category Mod-R with nice enough functorial properties. For example, torsion theory is a well suited incarnation of approximation theory. Of course, such an idea has been generalised to the additive setting itself, so that both Mod-R and other interesting categories related with R may be linked functorially. By the seminal work of Beilinson, Bernstein and Deligne (1982), the derived category of the ring turns out to admit useful torsion theories, called t-structures: they are pairs of full subcategories of D(R) whose intersection, called the heart, is always an abelian category. The so-called standard t-structure of D(R) has as its heart the module category Mod-R itself. Since then a lot of results devoted to the module theoretic characterisation of the hearts have been achieved, providing evidence of the usefulness of the t-structures in the representation theory of R. In 2020, following a research line promoted by many other authors, Saorin and Stovicek proved that the heart of any compactly generated t-structure is always a locally finitely presented Grothendieck categories (actually, this is true for any t-structure in a triangulated category with coproducts). Essentially, this means that the hearts of D(R) come equipped with a finiteness condition miming that one valid in Mod-R. In the present thesis we tackle the problem of characterising when the hearts of certain compactly generated t-structures of a commutative ring are even locally coherent. In this commutative context, after the works of Neeman and Alonso, Jeremias and Saorin, compactly generated t-structures turned out to be very interesting over a noetherian ring, for they are in bijection with the Thomason filtrations of the prime spectrum. In other words, they are classified by geometric objects, moreover their constituent subcategories have a precise cohomological description. However, if the ascending chain condition lacks, such classification is somehow partial, though provided by Hrbek. The crucial point is that the constituents of the t-structures have a different description w.r.t. that available in the noetherian setting, yet if one copies the latter for an arbitrary ring still obtains a t-structure, but it is not clear whether it must be compactly generated. Consequently, pursuing the study of the local coherence of the hearts given by a Thomason filtration, we ended by considering two t-structures. Our technique in order to face the lack of the ascending chain condition relies on a further approximation of the hearts by means of suitable torsion theories. The main results of the thesis are the following: we prove that for the so-called weakly bounded below Thomason filtrations the two t-structures have the same heart (therefore it is always locally finitely presented), and we show that they coincide if and only they are both compactly generated. Moreover, we achieve a complete characterisation of the local coherence for the hearts of the Thomason filtrations of finite length.

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