



UNIVERSITÀ
DI TRENTO

Dipartimento di
Matematica

DOTTORATO



CYCLE 34th
ORAL DEFENCE OF THE PHD THESIS

Thursday 13 October 2022 – at 3:00 pm
Seminar room “-1”

The event will take place in presence and online through the ZOOM platform.
To get the access codes please contact the secretary office

Lorenzo Martini
PhD Student in Mathematics

Local coherence of hearts in the derived category of a commutative ring

Abstract:

Approximation theory is a fundamental tool in order to study the representation theory of a ring R . Roughly speaking, it consists in determining suitable additive or abelian subcategories of the whole module category $\text{Mod-}R$ with nice enough functorial properties. For example, torsion theory is a well suited incarnation of approximation theory. Of course, such an idea has been generalised to the additive setting itself, so that both $\text{Mod-}R$ and other interesting categories related with R may be linked functorially. By the seminal work of Beilinson, Bernstein and Deligne (1982), the derived category of the ring turns out to admit useful torsion theories, called t -structures: they are pairs of full subcategories of $D(R)$ whose intersection, called the heart, is always an abelian category. The so-called standard t -structure of $D(R)$ has as its heart the module category $\text{Mod-}R$ itself. Since then a lot of results devoted to the module theoretic characterisation of the hearts have been achieved, providing evidence of the usefulness of the t -structures in the representation theory of R . In 2020, following a research line promoted by many other authors, Saorin and Stovicek proved that the heart of any compactly generated t -structure is always a locally finitely presented Grothendieck categories (actually, this is true for any t -structure in a triangulated category with coproducts). Essentially, this means that the hearts of $D(R)$ come equipped with a finiteness condition miming that one valid in $\text{Mod-}R$. In the present thesis we tackle the problem of characterising when the hearts of certain compactly generated t -structures of a commutative ring are even locally coherent. In this commutative context, after the works of Neeman and Alonso, Jeremias and Saorin, compactly generated t -structures turned out to be very interesting over a noetherian ring, for they are in bijection with the Thomason filtrations of the prime spectrum. In other words, they are classified by geometric objects, moreover their constituent subcategories have a precise cohomological description. However, if the ascending chain condition lacks, such classification is somehow partial, though provided by Hrbek. The crucial point is that the constituents of the t -structures have a different description w.r.t. that available in the noetherian setting, yet if one copies the latter for an arbitrary ring still obtains a t -structure, but it is not clear whether it must be compactly generated. Consequently, pursuing the study of the local coherence of the hearts given by a Thomason filtration, we ended by considering two t -structures. Our technique in order to face the lack of the ascending chain condition relies on a further approximation of the hearts by means of suitable torsion theories. The main results of the thesis are the following: we prove that for the so-called weakly bounded below Thomason filtrations the two t -structures have the same heart (therefore it is always locally finitely presented), and we show that they coincide if and only they are both compactly generated. Moreover, we achieve a complete characterisation of the local coherence for the hearts of the Thomason filtrations of finite length.

Supervisor: Francesca Mantese

CONTATTI

Staff di Dipartimento - Matematica
tel. 0461 281508-1625-1701-3786

phd.maths@unitn.it
www.maths.unitn.it