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ORAL DEFENCE OF THE PHD THESIS

### Wednesday 7 December 2022 – at 2.00 pm Seminar Room "-1"

The event will take place in presence and online through the ZOOM platform. To get the access codes please contact the secretary office

## Claudia De Lazzari

PhD Student in Mathematics

## Algebraic, geometric and numerical methods for Tensor **Network Varieties**

#### Abstract:

In this thesis, we study tensor network varieties, which are varieties of tensors described by the combinatorial structure of a given graph and two sets of integer weights, called bond and local dimensions, respectively. Tensor network varieties are geometric objects studied in the field of Algebraic Geometry, and they have received much attention in the recent years due to their usefulness in the field of Quantum Physics and other application areas. In the first part of the thesis, we study the dimension of tensor network varieties. We provide a completely general upper bound on their dimension and we give the exact value of the dimension in a particular range of parameters. We refine the upper bound in cases relevant for applications, such as matrix product states and projected entangled pairs states. We then focus on the study of the linear span of uniform matrix product states, which are translation invariant tensor network varieties associated to the cyclic graph. We provide nontrivial linear trace relations which prove the strict containment of the linear span in the ambient space as long as the number of sites is at least quadratic in the bond dimension, improving the state of the art. Finally, based on dimensional considerations, we propose a variation of the nonlinear conjugate gradient method used to approximate the ground states of a given Hamiltonian on the variety of matrix product states.

#### Supervisors:

Alessandra Bernardi (University of Trento) lacopo Carusotto (University of Trento)

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