# Detailed program of the course

# "Advanced numerical methods for environmental modeling"

May 15 – 30

*Room 1H of DICAM building, Via Mesiano 77, Trento*

## Week 1

**Monday, May 15, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Introduction to the course: course content, the DidatticOnline system, and information about the exam.

Hyperbolic and parabolic equations (examples, Euler, shallow water, Navier-Stokes, Richards equations). The linear scalar advection equation: exact solution of the Cauchy problem, method of characteristics, Eulerian and Lagrangian description, Riemann problem, Linear scalar advection equation with variable coefficients. Linear hyperbolic systems: Characteristic variables, Riemann problem for linear hyperbolic systems. Finite difference approximation of derivatives. The concept of Upwinding.

Afternoon (14:00 – 17:00) Coding session

Coding of finite difference methods for linear advection.
Introduction to Matlab. Practical implementation of various finite difference schemes for linear advection equations (Upwind scheme, Central scheme, Lax-Wendroff, Lax-Friedrichs) and linear hyperbolic systems. Extension to the 2D scalar advection with variable coefficients.

**Tuesday, May 16, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Accuracy and stability of numerical methods.
Analysis of errors: local truncation error, the modified equation method, and the von Neumann method of linear stability analysis.

Nonlinear scalar conservation laws.
The finite volume method. Exact solution of the Riemann problem for general scalar conservation laws. Shock waves and rarefaction fans. Newton's method for the solution inside the rarefaction fan. The integral form of the conservation law and finite volume method. The Godunov method.

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Riemann problem for the shallow water equations.
Systems of nonlinear conservation laws, Simple waves, Integral curves, Riemann invariants,

Rarefaction and shock waves for the shallow water equations, Assembling of the solution to the Riemann problem

Afternoon (14:00 – 17:00) Coding session

Practical implementation of the exact Riemann solver and of the Godunov finite volume method for nonlinear scalar conservation laws.
Implementation of the Godunov scheme for the 1D Shallow Water Equations with the exact Riemann solver.

**Wednesday, May 17, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Approximate Riemann solvers and TVD schemes.
HLL Riemann solver for the shallow water equations, Rusanov numerical flux. High-order reconstruction, 2-nd order in space method of Kolgan, 2-nd order space-time MUSCL-Hancock scheme

Afternoon (14:00 – 17:00) Coding session

Coding MUSCL-Hancock scheme for the 2D shallow water equations with the Rusanov numerical flux.

**Thursday, May 18, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Finite-volume methods for diffusion equation.
Energy conservation and the Fourier law, FTCS and BTCS schemes: stability analysis, local truncation error analysis.
Implementation of FTCS and BTCS schemes for the 1D heat equation
FTCS scheme for the linear heat equation. BTCS scheme: - use of the Thomas algorithm. Implementation of BTCS with Neumann boundary conditions - implementation of BTCS with Dirichlet boundary conditions - heterogeneous material properties - the conjugate gradient method (brief introduction, main properties, and efficient implementation) - the matrix-free conjugate gradient method, - the BTCS scheme based on the matrix-free conjugate gradient method

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Afternoon (14:00 – 17:00) Coding session

The FTCS and BTCS method for the 2D linear heat equation.
The heat equation in two space dimensions (2D): finite volume discretization based on the integral form and associated discrete heat fluxes, Implementation of the FTCS scheme, and the BTCS method for the 2D heat equation, matrix-free conjugate gradient method for the 2D problem with Neumann and Dirichlet type boundary conditions.

**Friday, May 19, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

BTCS and FTCS schemes for the Stefan problem (solidification).
BTCS scheme for the nonlinear parabolic equation. Nested Newton method of Casulli and Zanolli for mildly nonlinear algebraic systems. Implementation of the FTCS scheme for the 1D Stefan problem.

Afternoon (14:00 – 17:00) Coding session

Implementation of the 1D FTCS scheme and 2D BTCS scheme for the nonlinear parabolic PDEs and application to the Stefan problem: the nested Newton method of Casulli and Zanolli, choice of the initial guess, outer and inner iterations, comparison with the exact solution.

## Week 2

**Monday, May 21, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Semi-implicit method for 2D Incompressible Navier Stokes equations (INSE) on Cartesian staggered grids with FTCS and BTCS scheme for diffusion, transport of a passive scalar, and Boussinesq approximation for buoyancy,

Afternoon (14:00 – 17:00) Coding session

Coding: implementation of the SIMPLE method with FTCS and BTCS discretization of the viscous terms, advection-diffusion of a passive scalar.

**Tuesday, May 22, 2023** *Prof. Ilya Peshkov*

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Morning (9:00 – 12:00)

Continue the implementation of the SIMPLE method.

Afternoon (14:00 – 17:00) Coding session

Coupling the SIMPLE code with the semi-implicit code (Casulli-Zanolli method) for the Stefan system (solidification + convection)

**Wednesday, May 24, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Semi-Implicit methods for hyperbolic equations (the concept of an all-Mach number scheme). The idea of splitting a flux into explicit (non-stiff) and implicit (stiff) parts. The semi-implicit staggered scheme of Dumbser-Casulli for the Euler equations.

Afternoon(14:00 – 17:00) Coding session

Implementation of the 1D semi-implicit scheme of Dumber-Casulli for the Euler equations with the matrix-free conjugate gradient method for the pressure subsystem.

**Thursday, May 25, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00)

Discussion of the Semi-implicit method of Casulli for low-Froud number Shallow-Water Equations with wetting and drying conditions. Coupling of the SWE and sediment transport equations (Exner equations)

Morning (9:00 – 12:00) Coding session

Implementation of the method of Casulli for SWE with wetting and drying conditions, extension to SWE-Exner.

**Friday, May 26, 2023** *Prof. Annunziato Siviglia*

Morning (9:00 – 12:00)

To be announced

Afternoon (14:00 – 17:00) Coding session

To be announced

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## Week 3

**Monday, May 28, 2023** *Prof. Annunziato Siviglia*

Morning (9:00 – 12:00)

To be announced

Afternoon (14:00 – 17:00) Coding session

To be announced

**Tuesday, May 29, 2023** *Prof. Annunziato Siviglia*

Morning (9:00 – 12:00)

To be announced

Afternoon (14:00 – 17:00) Coding session

To be announced

**Wednesday, May 30, 2023** *Prof. Ilya Peshkov*

Morning (9:00 – 12:00) Coding session

Coupled nonlinear hyperbolic-nonlinear parabolic systems (Shallow Water+Richards=SWER) shallow water with a permeable bottom. Discussion of the semi-implicit discretization strategy and of the nested Newton's method of Casulli-Zanolli for mildly nonlinear systems and its realization for the Richards equation.

Afternoon (14:00 – 17:00)

Implementation of a semi-implicit scheme for the 2D SWER system (semi-implicit Cassuli scheme for the hyperbolic part + BTCS for the nonlinear parabolic)

Discussion of the projects for the exam.

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