



UNIVERSITÀ
DI TRENTO

Dipartimento di
Matematica

DOTTORATO



CYCLE 35th
ORAL DEFENCE OF THE PHD THESIS

Wednesday 10 May 2023 – at 14.30 pm

Department of Mathematics
Seminar Room 1

The event will take place in presence and online through the ZOOM platform.
To get the access codes please contact the secretary office

Enrico Savi

PhD Student in Mathematics

Global and local \mathbb{Q} -algebrization problems in real algebraic geometry

Abstract:

In 2020 Parusiński and Rond proved that every algebraic set $X \subset \mathbb{R}^n$ is homeomorphic to an algebraic set $X' \subset \mathbb{R}^n$ which is described globally (and also locally) by polynomial equations whose coefficients are real algebraic numbers. In general, the following problem was widely open:

Open Problem. *Is every real algebraic set homeomorphic to a real algebraic set defined by polynomial equations with rational coefficients?*

The aim of my PhD thesis is to provide classes of real algebraic sets that positively answer to above Open Problem. In my dissertation I will recall some new notions of real and complex algebraic geometry over \mathbb{Q} recently introduced by Fernando and Ghiloni. In particular, the main notion to outline is the so called $\mathbb{R}|\mathbb{Q}$ -regularity of points of a \mathbb{Q} -algebraic set $X \subset \mathbb{R}^n$. This definition suggests a natural notion of a \mathbb{Q} -nonsingular algebraic set $X \subset \mathbb{R}^n$. Then, I will explain the \mathbb{Q} -algebraic approximation techniques à la Akbulut-King developed in collaboration with Ghiloni and the main consequences we proved, that are, versions 'over \mathbb{Q} ' of the classical and the relative Nash-Tognoli theorems. In particular, we obtained a positive answer to above Open Problem in the case of compact nonsingular algebraic sets. Then, after extending 'over \mathbb{Q} ' the Akbulut-King blowing down lemma, we are in position to give a complete positive answer to above Open Problem also in the case of compact algebraic sets with isolated singularities. After algebraic Alexandroff compactification, we obtained a positive answer also in the case of non-compact algebraic sets with isolated singularities.

Supervisor: Riccardo Ghiloni

CONTATTI

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