



UNIVERSITÀ  
DI TRENTO

Dipartimento di  
Matematica

# DOTTORATO



CYCLE 35th  
ORAL DEFENCE OF THE PHD THESIS

**Monday 31 July 2023 – at 11.30 am**

Department of Mathematics  
Seminar Room 1

The event will take place in presence and online through the ZOOM platform.  
To get the access codes please contact the secretary office

## Daniele Volpe

PhD Student in Mathematics

### Paracausal deformations, Møller operators, and Hadamard states in CCR AQFT.

**Abstract:**

In this thesis, we address several problems related to the bosonic classical and algebraic quantum field theories in curved spacetime. In particular, the main question is: how do the theories change under finite global variations of the spacetime metric tensor? To answer this question a new deformation tool, the paracausal deformation, is developed and studied on its own as a new approach to investigate the structure of the space of globally hyperbolic metric tensors associated with a smooth manifold  $M$ . Then the classical Møller maps are constructed to compare solutions of the hyperbolic PDEs defining the classical field theories and the quantum Møller \*-isomorphisms follow to compare the CCR quantum algebras associated with the propagation of the quantum fields on the different background geometries. These maps possess the important property of preserving Hadamard states, providing a new way to implement the deformation argument used to prove the existence of such states in general globally hyperbolic spacetime. Moreover, the algebraic quantization of the Proca field, i.e the massive spin 1 field, on a general globally hyperbolic spacetime is for the first time studied in detail: by employing techniques coming from microlocal analysis and spectral theory a Hadamard state is constructed on ultrastatic spacetimes and then the Møller operator is used to prove the existence of such states in general globally hyperbolic spacetimes. A discussion about the definition of Hadamard states for the massive vector fields closes the work.

**Supervisor:** Valter Moretti, Università di Trento

**Co-supervisor:** Simone Murro, Università di Genova

**CONTATTI**

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