Abstract:
In this Thesis we explore, both theoretically and numerically, splitting strategies for a hyperbolic system of one-dimensional (1D) blood flow equations with a passive scalar transport equation. Our analysis involves a two-step framework that includes splitting at the level of partial differential equations (PDEs) and numerical methods for discretizing the ensuing problems. This study is inspired by the original flux splitting approach of Toro and Vázquez-Cendón (2012) originally developed for the conservative Euler equations of compressible gas dynamics. In this approach the flux vector in the conservative case, and the system matrix in the non-conservative one, are split into advection and pressure terms: in this way, two systems of partial differential equations are obtained, the advection system and the pressure system.

In the first part of the Thesis a detailed theoretical analysis is presented, involving the exact solution of the Riemann problem for the 1D blood flow equations, depicted for a general constant momentum correction coefficient and a tube law that allows to describe both arteries and veins with continuous or discontinuous mechanical and geometrical properties and with an advection equation for a passive scalar transport added. In addition, we propose a procedure to compute the obtained exact solution and finally we validate it numerically, by comparing exact solutions to those obtained with well-known, numerical schemes on a carefully designed set of test problems. Furthermore, an analogous theoretical analysis and resolution algorithm are presented for the advection system and the pressure system arising from the splitting at the level of PDEs of the complete system of 1D blood flow equations.

In the second part of the Thesis we present novel finite volume-type, flux splitting-based, numerical schemes for the conservative 1D blood flow equations and splitting-based numerical schemes for the non-conservative 1D blood flow equations that incorporate an advection equation for a passive scalar transport, considering arteries and veins and taking into account a general constant momentum correction coefficient. A detailed efficiency analysis is performed in order to showcase the advantages of the proposed methodologies in comparison to standard approaches.